

UNIVERSITY OF TECHNOLOGY SYDNEY

**Uncertainty Modelling and Motion
Planning of an Inchworm Robot
Navigating in Complex Structural
Environments**

by

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A thesis submitted in partial fulfillment for the
degree of Doctor of Philosophy

in the
Faculty of Engineering and IT
Intelligent Mechatronic Systems Group

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Declaration of Authorship

I, David Pagano , declare that this thesis titled, ‘Uncertainty Modelling and Motion Planning of an Inchworm Robot Navigating in Complex Structural Environments’ and the work presented in it are my own. I confirm that:

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Abstract

Faculty of Engineering and IT
Intelligent Mechatronic Systems Group

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Many ferromagnetic structures require continuous inspection and maintenance routines to ensure longevity, structural integrity and aesthetics. For most structures, routines are performed by teams of personnel, with each individual performing specific tasks. These tasks may be highly hazardous; being performed at height, in confined spaces or in the presences of hazardous materials such as lead based paints and vehicle fumes. Adopting a robotic solution for inspections would significantly improve occupational health and safety for maintenance personnel, while increasing the quality and reducing the cost.

An inchworm robot has been developed for inspection of confined spaces in the Sydney Harbour Bridge. With a 7 degree of freedom multi-link serial body and magnetic pads for adhesion, the inchworm robot provides a dexterous means for climbing and inspecting particularly difficult-to-access sections of the bridge. However, due to the structure and the adhesion mechanism of the inchworm type robot, deformation of the robot body (i.e. structural uncertainty) and inaccurate landing position of the permanent magnet adhesion pads (i.e. hand position uncertainty) cause imperfect knowledge about the robot state. This prevents safe motion in a real world setting. The combination of these uncertainties present a unique challenge in robot motion planning and collision avoidance which is not considered in the literature.

This thesis first focuses on developing a model for representing the structural and hand position uncertainties. The model describes the uncertainty in the coordinate frame of reference for the joints.

A 3D probabilistic force field (3D-PF²) algorithm is developed to incorporate the uncertainties and allow for smooth, collision-free path planning. A force field surrounds each link to prevent collisions with each force field sized to account for the dimensions of the link and the uncertainty at the joints related to the link. Force fields are used to generate repulsive forces which push the robot away from obstructions while an attractive force pulls the end-effector towards a goal location.

A Line of Sight Tree (LoST) algorithm is developed for longer time-horizon motion planning with the 3D-PF² algorithm used for local motion planning. The LoST algorithm provides waypoints as goal locations for the 3D-PF² algorithm. Waypoints are found in a

manner loosely based on the way a person views a scene whereby their gaze tends towards important regions such as the edges of objects.

Extensive simulations and experiments have been conducted to test the performance of both the 3D-PF² and the LoST algorithms within a number of environments including the specific application scenario at the Sydney Harbour Bridge.

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Abbreviations

3D-F²	3 Dimensional Force Field
3D-PF²	3 Dimensional Probabilistic Force Field
COG	Centre of Gravity
DOF	Degrees of Freedom
EKF	Extended Kalman Filter
FEA	Finite Element Analysis
IMU	Inertial Measurement Unit
HMI	Human-Machine Interface
GA	Genetic Algorithm
LOS	Line of Sight
LoST	Line of Sight Tree
LQG	Linear-quadratic Gaussian
MDP	Markov Decision Process
NSW	New South Wales (Australia)
PF	Potential Field
POMDP	Partially Observable Markov Decision Process
PRM	Probabilistic Roadmap
RANSAC	Random sample consensus
RGB-D	Red, green, blue and depth
RRT	Rapidly-exploring Random Tree
SLAM	Simultaneous Localisation and Mapping
UTS	University of Technology Sydney
VFF	Virtual Force Field

Nomenclature

General Formatting Style

$f(\dots)$	A scalar valued function
$\mathbf{f}(\dots)$	A vector valued function
$[\dots]^T$	Transpose
$ \cdot $	Absolute value
$\ \cdot\ $	Vector length and normalised vector

Local and Global Variables

ψ_G	The global coordinate frame
ψ_R	Coordinate frame of reference of a link
i, j, k	Variables signifying the index of and counts associated with joints and links

Specific Symbol Usage

${}^{i-1}T_i$	A homogeneous transformation matrix (4 by 4)
a	A DH parameter describing the distance along the x-axis
d	A DH parameter describing the distance along the z-axis
α	A DH parameter describing the rotation about the z-axis
\mathbf{R}	A rotation matrix (3 by 3)
$\mathbf{R}_n, \mathbf{R}_a, \mathbf{R}_o$	The normal, approach and orientation vectors of a rotation matrix (3 by 1)
n_x, n_y, n_z	Components of normal vector
a_x, a_y, a_z	Components of approach vector
o_x, o_y, o_z	Components of orientation vector

\mathbf{P}	A position vector (3 by 1)
x, y, z	Components of the position vector along a x-, y- and z-axis
\mathbf{P}_{xy}	A position vector along the xy-plane (2 by 1)
n	Number of joints in the multi-link serial robot
\mathbf{q}	A joint position matrix (n by 1)
q	A component of the joint position matrix
${}^0\mathbf{P}_g$	Goal location position vector (3 by 1)
${}^0\mathbf{R}_g$	Goal location orientation vector (3 by 1)
l_c	Distance between the base and the end-effector
E_c	Allowed maximum structural translational deformation
γ_c	Allowed maximum structural rotational deformation
d_{xy}	Maximum distance along the xy-plane between a joint's and a preceding joint's coordinate frames of reference
β_c	Joint distance ratio
$G_c(\dots)$	Function relating allowed maximum translational deformation to the joint distance ratio
$F_c(\dots)$	Function relating allowed maximum rotational deformation to the joint distance ratio
$\mu_{E.i}$	Estimate of the mean structural translational deformation at a joint
$\mu_{\gamma.i}$	Estimate of the mean structural rotational deformation at a joint
$\sigma_{E.i}$	Structural translational variance
$\boldsymbol{\sigma}_{E.i}$	A matrix describing the structural translational variance in 3D (3 by 3)
$\mu_{x.h}, \mu_{y.h}$	Hand position mean along the x- and y-axis
$J_c(\dots)$	Function relating the hand position variance to the joint position ratio
d_a, d_b, d_e	Preparatory approach, surface approach and end-effector distances to the surface goal location

K_g	The growth factor
σ_h	Hand position variance
$\boldsymbol{\sigma}_h$	A matrix describing the hand position variance in 3D (3 by 3)
$\boldsymbol{\sigma}$	A matrix describing the variance at the joint (3 by 3)
\mathcal{T}	A homogeneous transformation matrix with uncertainty (4 by 4)
\mathcal{R}	A rotation matrix with uncertainty (3 by 3)
\mathcal{P}	A position vector with uncertainty (3 by 1)
\mathbf{I}	Mass-inertia matrix of the robot links
$\boldsymbol{\beta}$	Damping coefficient matrix of the robot joints
\mathbf{k}_{sp}	Stiffness coefficient matrix of the robot joints
$\boldsymbol{\Gamma}$	A torque-force matrix defining in joint space (n by 1)
τ	Joint torque-force of a joint, a component in $\boldsymbol{\Gamma}$
$\dot{\mathbf{q}}$	A joint velocity matrix (n by 1)
$\ddot{\mathbf{q}}$	A joint acceleration matrix (n by 1)
\mathbf{H}	A force transformation matrix
\mathbf{h}	Component of the force transformation matrix, H
\mathbf{M}	A rotation skew-symmetric matrix
\mathbf{L}	Configuration matrix of a joint
$l_{r.x}, l_{r.y}, l_{r.z}$	Rotational components of the configuration matrix of a joint
$l_{t.x}, l_{t.y}, l_{t.z}$	Translational components of the configuration matrix of a joint
\mathcal{H}	A probabilistic force transformation matrix
\mathcal{M}	A probabilistic rotation skew-symmetric matrix
$\hat{\mathbf{h}}$	Component of the probabilistic force transformation matrix, H
S	A skew-symmetric matrix (3 by 3)
\mathbf{F}	A 6DOF spatial force acting on the multi-link serial manipulator
\mathbf{f}	The positional component of the spatial force
$\boldsymbol{\omega}$	The rotational component of the spatial force

\mathcal{F}	A 6DOF spatial force acting on the probabilistic multi-link serial manipulator
\mathbf{f}	The positional component of the probabilistic spatial force
\mathbf{w}	The rotational component of the probabilistic spatial force
P_{att}	The point the attractive force is applied
K_{att}	Coefficient of an attractive force amplitude
K_s	A constant for defining a transient state of the attractive force
K_{zero}	A small non-zero positive constant
K_P	Ellipsoid coverage constant
K_f	Coefficient of a repulsive force amplitude
K_{de}	The environmental repulsive force distance factor
K_{ds}	The self repulsive force distance factor
d_0	Distance between the closest points between two links
E_r	Force field error factor
$\hat{\mathbf{D}}$	The variance force field
$\hat{\mathbf{D}}_{min}$	The minimum force field
$\hat{\mathbf{D}}_{minV}$	The minimum variance force field
$\hat{\mathbf{D}}_{maxV}$	The maximum variance force field
\mathbf{R}_{ff}	A rotation matrix for a force field (3 by 3)
\mathbf{r}	A matrix describing the radius of a force field (3 by 1)
\mathbf{c}	A matrix describing the centre of a force field (3 by 1)
ξ_{min}, ξ_{max}	The minimum and maximum variance factors
\mathbf{V}	A matrix of a component's vertices
$\sigma_{\mathbf{V}}$	A matrix of the variance of a component's vertices
$\mathbf{P}_{\sigma_{\mathbf{V}}}$	A matrix of points representing the variance of a component's vertices
$\hat{\mathbf{V}}$	A component's vertices relative to a unit sphere
$\psi_{\hat{\mathbf{D}}}$	A matrix used to create a unit sphere
ξ_{dist}	Furthest distance to a vertex within the unit sphere
\mathbf{r}_{min}	The radius of the minimum force field

\mathbf{r}_{minV}	The radius of the minimum variance force field
\mathbf{r}_{maxV}	The radius of the maximum variance force field
\mathbf{P}_{obs}	The environment voxels
$\hat{\mathbf{P}}_{obs}$	The environmental voxels transformed relative to a unit sphere
$\hat{\mathbf{P}}_{dist}$	The distance from the origin of the unit sphere to the transformed environmental voxels
\hat{r}_l	The radius of the maximum variance unit sphere
d_{cls}	The distance penetrated into a force field
k_{sample}	Local minima detection sample period
k_q	Insignificant motion detection threshold
$k_{\dot{q}}$	Insignificant velocity detection threshold
$k_{q_{set}}$	Repeated motion threshold
\mathbf{N}_0	The start node
\mathbf{N}_g	The goal node
$\mathbf{N}, \mathbf{N}_{CR}$	An intermediate node and a common region centre
u	Current nodes index
v	Parent node index
\mathbf{P}_{mid}	A discontinuous minpoint
$\mathbf{P}_{min}, \mathbf{P}_{max}$	The end point of the shorter and longer discontinuous pair
l_{min}, l_{max}	The shorter and longer lengths of a discontinuous pair
θ_{min}	The minimum angle
Z_{off}	Number of rays in the minimum angle
θ_{res}	The angle between rays
c	The radius of a robot's workspace
\mathbf{L}	A matrix of ray distances
δ	A ray's offset angle
$\boldsymbol{\theta}$	A matrix of angles between the each ray and the offset ray

m	Weighting criteria set
d_m	The distance for a weighting criteria set
K_m	Normalised distance for a weighting criteria set
α_m	The weighting factor for a weighting criteria set
w	The final weight for an intermediate node or common region centre

Glossary of Terms

Base	The foot pad of the robot that is currently fixed to the surface.
Common Region	Where visible regions of the end node and an intermediate node overlap. Termination criteria.
Common Region Centre	The midpoint of the common region.
End Node	Ending position of the inchworm robot end-effector.
Environment	A 3D structure in which a manipulator is positioned. Assumed to have some structural characteristics such as planar surfaces.
Ferromagnetic	Made of metals to which magnets are attracted.
Hand position uncertainty	The deviation in the end-effector position when attaching to the surface caused by the magnetic force.
Hybrid Planner	Path planner which comprises of two or more individual path planners working in tandem.
Inchworm Robot	A 7DOF serial robot which uses magnets to adhere to ferromagnetic surfaces.
Intermediate Nodes	Points within Cartesian space which serve as subgoals with the LoST algorithm.
Line of sight	The visibility of the surroundings from a point which is not impeded by obstacles.
Maximum index level	Maximum number of iterations down a single branch of the line of sight tree the line of sight algorithm searches.

Multi-link serial robot	A series of actuated joints and connected links describing a robot from a base to an end-effector.
Node	Possible robot position in Cartesian space.
Obstacle	An object which a manipulator can potentially collide with.
Planning	The act of generating a path (and motion) which the robot can then follow between two poses.
Pose	The joint configuration of a robot.
Ray Casting	Process which uses rays to determine intersection points with obstacles.
Start Node	The start position of the inchworm robot end-effector.
Start Pose	The start pose of the inchworm robot.
Structural uncertainty	The deformation caused by gravitational loads on the inchworm robot.
Surface Normal	A 3D vector perpendicular to a surface.
Visible Region	Area surrounding a node which is visible and determined through line of sight.
Voxel	Volumetric Pixel which represents a 3D cube-like volume in Cartesian space.
Waypoint	Either a common region centre or intermediate node used as a goal for the 3D-PF ² algorithm to manoeuvre to.